Part I

Introductory Session: SH 2–3, BM 2

Outline

- Elementary algebra
- Equations
- Summation notation

Real numbers

- $\mathbb{N}$ (natural numbers): 1,2,3
- $\mathbb{Z}$ (integers): 0,1,-1
- $\mathbb{Q}$ (rational numbers): a/b, where $a, b \in \mathbb{Z}, b \neq 0$
  - decimal system: $42.14 = 4 \cdot 10^1 + 2 \cdot 10^0 + 1 \cdot 10^{-1} + 4 \cdot 10^{-2}$
  - scientific notation: `2.34e-02`$= 2.34 \cdot 10^{-2} = 0.0234$
    Note: Here, `e` is not the Euler constant!
  - can be written as *finite* decimal fractions (see above) or *periodic* decimal fractions:
    $13/11 =$ 1.18181818
- $\mathbb{R}$ (real numbers): *arbitrary* decimal fractions – all of the above, plus:
  $\sqrt{2}, \pi, 2^{\sqrt{2}}, 0.12112111211112\ldots$ and many (many) others

Integer Powers I

Definition:

$$a^n := \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}$$

$$a^0 := 1 \text{ for } a \neq 0$$

$$a^{-n} := \frac{1}{a^n} \text{ for } a \neq 0$$

Properties:

$$a^r \cdot a^s = a^{r+s}$$

$$(a^r)^s = a^{rs}$$

Integer Powers II

Example

If $x^{-2}y^3 = 5$, compute $x^2y^{-3} + 2x^{-10}y^{15}$

(x^-2y^3)^5 / 5^5

0.2+2*3125= 6250.2

Integer Powers III

Example

Suppose you deposit €1000 in a bank account paying 2% interest at the end of each year. How much do you have after 5 years?

1000*1.02^5

Integer Powers IV

Example

Suppose you buy something for €$1000 \cdot 1.02^5$ which decreases in value (depreciates) by 2% per year. How much is it worth after 5 years?

1000*1.02^5*0.98^5=998

Integer Powers V

Example

How much money should you have deposited in a bank 5 years ago at 2% yearly interest in order to buy something for €1000 today?

$$x*1.02^5=1000$$

$$x=1000/1.02^5$$

x=—905.73

Some important rules of algebra I

$$
\begin{aligned}
-(a+b) &= -a \; - \; b \\
a(b+c) &= ab + \text{ac} \\
(a+b)(c+d) &= ac + ad + bc + bd \\
(a+b)^2 &= \text{a^2+2ab +b^2} \\
(a-b)^2 &= \text{a^2-2ab+b^2} \\
(a+b)(a-b) &=
\end{aligned}
$$

Some important rules of algebra II

Example

Expand and simplify: $(2t-1)(t^2-2t+1)$

Some important rules of algebra III

Example

Expand and simplify: $(a+1)^2+(a-1)^2-2(a+1)(a-1)$

a^2+2a+1+a^2-2a+1-2(a^2-1)=4

Some important rules for fractions I

$$\frac{a \cdot \cancel{c}}{b \cdot \cancel{c}} = \frac{a}{b} \qquad \text{if } b \neq 0 \text{ and } c \neq 0$$

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Some important rules for fractions II

Example

Simplify:

$$\frac{1}{1+x^{p-q}}+\frac{1}{1+x^{q-p}}$$

Fractional powers I

Find x such that $x^2 = a$?

$$a^{1/2} \cdot a^{1/2} = a^{1/2+1/2} = a^1 = a$$

Find x such that $x^n = a$?

$$a^{1/n} \cdot \ldots \cdot a^{1/n} = a^{1/n+\ldots+1/n} = a^1 = a$$

Notation:

$$a^{1/2} = \sqrt{a}$$

$$a^{1/n} = \sqrt[n]{a}$$

Fractional powers II

Example

Compute $\sqrt[3]{27}$, $(1/32)^{1/5}$, and $0.0001^{0.25}$.

Fractional powers III

Example

An amount of €5000 in an account has increased to €6000 in 20 years. What (constant) yearly interest rate p has been used?

5000*(1+p)^20= 6000

(1+p)^20=1.2

1+p=20th root of 1.2-1=0.0092

Inequalities I

Example

Find what values of x satisfy $3x - 5 > x - 3$.

$$2x>2$$
$$x>1$$

Inequalities II

Example

Find all x such that $|3x-2| \leq 5$.

2 solutions

+-

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Equations I

Equations are called *equivalent* if they have the same solutions, and

- adding/subtracting the same number
- multiplying (or dividing) by the same number $\neq 0$

to both sides of the equality sign constitutes an equivalence transformation. We often call doing fancy equivalence transformations *solving*.

Equations II

Example

Solve for x:

$$\sqrt{1+x} + \frac{ax}{\sqrt{1+x}} = 0$$

1+x +ax=0

Equations III

x(1+a)=-1

x=-1/1+a

Example

A firm manufactures a commodity that costs €20 per unit to produce. In addition, the firm has fixed costs of €2000. Each unit is sold for €75. How many units must be sold if the firm is to meet a profit target of €14 500?

c(x)= 2000+20x

r(x)=75x

p(x)= r(x)-c(x)

75x-(2000+20x)=14500

55x=16500

x=300

Quadratic equations I

Find x, such that

$$ax^2 + bx + c = 0, \qquad \text{where } a, b, c \in \mathbb{R}$$

- Easy case 1: $a = 0$: bx+c+0, x=-c/b, b is not 0
- ax^2+c=0, x=+- root-c/ a a is not 0, c/a is less or equal to 0

  Easy case 2: $b = 0$:
- Easy case 3: $c = 0$:

ax^2 +bx = x(ax+b)=0

Quadratic equations II

General case: If $b^2 - 4ac \geq 0$ and $a \neq 0$, then

$$ax^2 + bx + c = 0 \quad \text{if and only if} \quad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Quadratic equations III

Example

A producer faces the following demand: $P = 100 - 2Q$, where P stands for the price of a certain product and Q for the quantity of products sold. For what price is the total revenue $TR = P \cdot Q$ equal to zero?

(100-2q)*q= 100q-2q^2=0

q1=0

q2-50

p1=100

p2=0

Outline

- Elementary algebra
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Summation notation I

$$\sum_{i=1}^{n} N_i := N_1 + N_2 + \ldots + N_n$$

Some important properties:

$$\sum_{i=1}^{n} (a_i + b_i) = \boxed{\text{Text}}$$

$$\sum_{i=1}^{n} c \cdot a_i = \boxed{}$$

Summation notation II

Example

Evaluate

$$\sum_{i=-2}^{3} (i+3)^i$$

Text

Summation notation III

Example

Express in summation notation:

$$1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \ldots - \frac{x^{79}}{80} + \frac{x^{80}}{81}$$

(x^i-1)
/i)